Preempt a Job or Not in EDF Scheduling of Uniprocessor Systems

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APPENDIX

In this section, we prove Lemma 2: the time-complexity of Theorem 1 for a given task set with given $\{X_i\}$ is pseudo-polynomial in the task parameters, if $\sum_{\tau_i \in \mathcal{T} | X_i = 0} C_i/T_i + \sum_{\tau_i \in \mathcal{T} | X_i = 1} (C_i + \alpha)/T_i$ is upper-bounded by a constant that is strictly smaller than 1.

We first prove that we need not test Eq. (9) for some l larger than a certain value. To do this, we present a relevant property of the fp-EDF analysis without any preemption delay [4] as follows:

Lemma 6: (Theorem 6 in [28]) Suppose $U < 1$ holds and the condition for the fp-EDF analysis without any preemption delay [4] (i.e., Eq. (2) in this paper) is violated for some $l > 0$. Then, the condition should be also violated for some $0 < l \leq l_{max} \triangleq \max\Big(\max_{\tau_i \in \mathcal{T}} (D_i - T_i), \sum_{\tau_i \in \mathcal{T}} (T_i - D_i) \cdot U_i/(1-U) \Big)$, where $U_i \triangleq C_i/T_i$ and $U \triangleq \sum_{\tau_i \in \mathcal{T}} U_i$.

The lemma implies that we need to test Eq. (2) only for $0 < l \leq l_{max}$. Then, using Lemma 6, we can upper-bound l for Theorem 1 as follows.

Lemma 7: Suppose $U' < 1$ holds and Eq. (9) is violated for some $l > 0$. Then, the condition is also violated for some $0 < l \leq l'_{max} \triangleq \max\Big(D_n, \max_{\tau_i \in \mathcal{T}} (D_i - T_i), \sum_{\tau_i \in \mathcal{T}} (T_i - D_i) \cdot U'_i/(1-U') \Big)$, where $U'_i \triangleq C_i/T_i$ for $X_i = 0$ and $U'_i \triangleq (C_i + \alpha)/T_i$ for $X_i = 1$, and $U' \triangleq \sum_{\tau_i \in \mathcal{T}} U'_i$.

Proof: Consider a new task set $\mathcal{T}'$ in which all task parameters are the same as $\mathcal{T}$ but the execution time of each τ_i with $X_i = 1$ is $C_i + \alpha$. We consider two cases: (i) Eq. (2) for $\mathcal{T}'$, and (ii) Eq. (9) for $\mathcal{T}$. Since B in Eq. (7) is always equal to zero when $l \geq D_n$, testing (i) is exactly the same as testing (ii) for $l \geq D_n$. For $l < D_n$, testing (i) is special case of testing (ii), i.e., testing (i) is the same as testing (ii) with $b = 0$.

By Lemma 6, we guarantee that if (i) is violated for $l \geq D_n$, (i) is also violated for $l < D_n$. Since (ii) with $b = 0$ is checked, the lemma holds. □

So far, we derived an upper-bound of l to be checked; by Lemma 7, we need to test Eq. (9) only for $l \leq l'_{max}$. To further reduce the number of candidates of l to be checked, we paraphrase Theorem 1 as follows. *A task set $\mathcal{T}$ is schedulable by cp-EDF on a uniprocessor platform in the presence of the preemption delay α, if the following condition holds:*

$$\max_{l>0} \left\{ \frac{\text{LHS of Eq. (9)}}{l} \right\} \leq 1. \tag{10}$$

In order to utilize the alternative form of Theorem 1 for less number of candidates of l to be checked, we derive the following lemma.

Lemma 8: The LHS of Eq. (10) is maximized when l or $l - b$ belongs to $\Omega \triangleq \{D_i + n \cdot T_i | \tau_i \in \mathcal{T}, n = 0, 1, 2, \cdots\}$.

Proof: Suppose that the LHS of Eq. (10) is maximized even though neither l nor $l-b$ belongs to Ω. Let l_1 and b_1 denote l and b when the LHS of Eq. (10) is maximized. We show a contradiction.

We consider $l = l_1 - \epsilon$, where ϵ is a sufficiently small value. Since both l_1 and $l_1 - b_1$ do not belong to Ω, the following inequalities hold for every $\tau_i \in \mathcal{T}$: $\mathsf{DBF}(\tau_i, l_1 - b_1) = \mathsf{DBF}(\tau_i, l_1 - \epsilon - b_1)$, $\mathsf{DBF_p}(\tau_i, l_1 - b_1) = \mathsf{DBF_p}(\tau_i, l_1 - \epsilon - b_1)$, and $\mathsf{DBF}(\tau_i, l_1) = \mathsf{DBF}(\tau_i, l_1 - \epsilon)$. Therefore, the LHS of Eq. (9) for $l = l_1 - \epsilon$ is the same as that for $l = l_1$, but $l_1 - \epsilon$ itself is smaller than l_1. This means that (LHS of Eq. (9))/l for $l = l_1 - \epsilon$ is larger than that for $l = l_1$, which a contradiction. □

Then, Lemma 8 indicates that we need to test Eq. (9) only for l such that l or $l - b$ belongs to Ω. Combining Lemmas 8 and 7 together, we know that the number of candidates of l (and $l - b$) to be checked is $O(\sum_{\tau_i \in \mathcal{T}} l'_{max}/T_i)$.

The remaining step is to upper-bound the number of b to be checked for given l or $l - b$. Since we assume a quantum-based time as mentioned in Section 2.1, an upper-bound of the number is $O(\max_{\tau_i \in \mathcal{T}} C_i)$, which is an upper-bound of B in any case.

Since calculating LHS of Eq. (9) for a given task set with given $\{X_i\}$ and a given l and b requires $O(n)$, the total time-complexity of testing Theorem 1 for a given task set with given $\{X_i\}$ is $O(P)$, where

$$P = n \cdot \max_{\tau_i \in \mathcal{T}} C_i \cdot \sum_{\tau_i \in \mathcal{T}} l'_{max}/T_i. \tag{11}$$

Similar to the fp-EDF analysis without any preemption delay [4] (i.e., Eq. (2) in this paper), the total time-complexity is pseudo-polynomial in the task parameters, if U' is upper-bounded by a constant that is strictly smaller than 1. Note that the total time-complexity derived here is a rough but safe upper-bound, and we can further reduce the time-complexity by applying a technique to investigate l more efficiently in [28].